

Ratios

A ratio is a comparison of two quantities that have the *same units*. You can express a ratio in any one of the following ways:

$$\frac{18}{5} \qquad 18:5 \qquad 18 \text{ to } 5$$

Example #1: If one store has 360 items and another store has 100 of the same items, express the ratio of the items.

$$\frac{360}{100} \qquad \text{or} \qquad 360:100 \qquad \text{or} \qquad 360 \text{ to } 100$$

Ratios are usually written in lowest terms; therefore, the above example would reduce in this way:

$$\frac{360}{100} \div 20 \qquad (\text{What is the largest number you can divide both values by?})$$

$$\frac{18}{5}$$

Example #2: John earns \$350 a week. His take-home pay, however, is \$295. What is the ratio of his gross pay to his take-home pay.

$$\frac{350}{295} = \frac{70}{59}$$

Rates

A rate is a comparison of two quantities that have *different units*. Rates are usually expressed in the fractional form.

Example: Francine paid \$16 for her 12-month subscription to *Better Homes and Gardens* magazine. Express as a rate.

$$\frac{\$16.00}{12 \text{ magazines}} = \frac{\$4.00}{3 \text{ magazines}}$$

If Francine wants to know how much she pays for each (1) magazine, she can divide \$4 by 3 magazines. This will give her the price per magazine (also called the **unit rate**).

$$\frac{\$4.00}{3} = \$1.33/\text{magazine}$$

KeyConcept Cross Products Property

Words	In a proportion, the product of the extremes equals the product of the means.
Symbols	If $\frac{a}{b} = \frac{c}{d}$ when $b \neq 0$ and $d \neq 0$, then $ad = bc$.
Example	If $\frac{4}{10} = \frac{6}{15}$, then $4 \cdot 15 = 10 \cdot 6$.

$$\begin{array}{ccc} \text{extreme} \rightarrow & \frac{a}{b} = \frac{c}{d} & \leftarrow \text{mean} \\ & \frac{b}{d} = \frac{c}{a} & \\ \text{mean} \rightarrow & & \leftarrow \text{extreme} \end{array}$$

KeyConcept Equivalent Proportions

Symbols	The following proportions are equivalent.
	$\frac{a}{b} = \frac{c}{d}, \frac{b}{a} = \frac{d}{c}, \frac{a}{c} = \frac{b}{d}, \frac{c}{a} = \frac{d}{b}$
Examples	$\frac{28}{50} = \frac{x}{755}, \frac{50}{28} = \frac{755}{x}, \frac{28}{x} = \frac{50}{755}, \frac{x}{28} = \frac{755}{50}$.

Proportions

A proportion is a statement that two ratios or rates are equal. It can be given as a sentence in words, but most often a proportion is an algebraic equation.

The arithmetic equation $\frac{3}{5} = \frac{21}{35}$ is a proportion because its cross products are equal.

$$3 \times 35 = \mathbf{105} \quad \text{and} \quad 5 \times 21 = \mathbf{105}$$

Proportions are solved by using this cross-product rule.

Example #1: $\frac{4}{9} = \frac{x}{36}$

$$4 \times 36 = 9x$$

$$144 = 9x$$

$$\frac{144}{9} = x$$

$$16 = x$$

Example #2: $\frac{72}{1.5} = \frac{12}{x}$

$$72x = 1.5 \times 12$$

$$72x = 18$$

$$x = \frac{18}{72}$$

$$x = .25 \text{ or } \frac{1}{4}$$

Applied Proportion Problems

Many problems can be solved by setting up a **direct proportion** (an increase in one quantity leads to a proportional increase in the other quantity) or by setting up **equivalent rates**.

Example: In one day you earn \$75 for 8 hours of work. If you work 37.5 hours for the week, what will your weekly pay be?

$$\frac{8 \text{ hours}}{37.5 \text{ hours}} = \frac{\$75}{x}$$

$$8x = 75 \times 37.5$$

$$8x = 2812.5$$

$$x = \frac{2812.5}{8}$$

$$x = \$351.56$$

$$\frac{8 \text{ hours}}{\$75} = \frac{37.5 \text{ hours}}{x}$$

$$8x = 75 \times 37.5$$

$$8x = 2812.5$$

$$x = \frac{2812.5}{8}$$

$$x = \$351.56$$

or

7.1 HOMEWORK RATIOS AND PROPORTIONS

A. Write each ratio as a fraction in lowest terms.

1. 2 to 4

6. 3 to 12

11. 35:7

2. $\frac{15}{20}$

7. 7:4

12. $\frac{8}{28}$

3. 6:18

8. $\frac{18}{12}$

13. 24 to 96

4. 21:15

9. 20:16

14. 9:27

5. $\frac{12}{18}$

10. 15 to 36

15. $\frac{11}{88}$

B. Write each of the following rates as a unit rate.

1. $\frac{3 \text{ Tbsp}}{2 \text{ tsp}}$

2. $\frac{135 \text{ pitches}}{45 \text{ strikes}}$

3. $\frac{128 \text{ miles}}{4 \text{ hours}}$

4. $\frac{2250 \text{ pencils}}{18 \text{ boxes}}$

5. $\frac{\$450}{18 \text{ shares}}$

6. $\frac{2500 \text{ meters}}{15 \text{ seconds}}$

7. $\frac{\$5,082}{475 \text{ sq.yds.}}$

8. $\frac{750 \text{ gallons}}{14 \text{ minutes}}$

C. Solve each proportion and give the answer in simplest form.

1. $6 : 8 = n : 12$

2. $\frac{2}{7} = \frac{8}{n}$

3. $\frac{n}{6} = \frac{11}{3}$

4. $4 : n = 6 : 9$

5. $\frac{3}{n} = \frac{2}{5}$

6. $\frac{0.4}{1.5} = \frac{12}{n}$

7. $2\frac{1}{2} : 3\frac{1}{2} = n : 2$

8. $1 : 2 = n : 9$

9. 4 to 8 = 15 to n

10. $18 : n = 3 : 11$

11. $\frac{5}{6} = \frac{n}{30}$

12. $\frac{12}{40} = \frac{n}{25}$

13. $8 : 19 = 14 : n$

14. $\frac{10}{n} = \frac{2}{1.7}$

15. $24 : \frac{1}{4} = n : \frac{1}{3}$

16. 44 to 121 = n to 11

D. Solve by using a proportion. Round answers to the nearest hundredth if necessary.

1. You jog 3.6 miles in 30 minutes. At that rate, how long will it take you to jog 4.8 miles?

2. You earn \$33 in 8 hours. At that rate, how much would you earn in 5 hours?

3. An airplane flies 105 miles in $\frac{1}{2}$ hour. How far can it fly in $1\frac{1}{4}$ hours at the same rate of speed?

4. What is the cost of six filters if eight filters cost \$39.92?
5. If one gallon of paint covers 825 sq. ft., how much paint is needed to cover 2640 sq. ft.?
6. A map scale designates 1" = 50 miles. If the distance between two towns on the map is 2.75 inches, how many miles must you drive to go from the first town to the second?
7. Bob is taking his son to look at colleges. The first college they plan to visit is 150 miles from their home. In the first hour they drive at a rate of 60 mph. If they want to reach their destination in $2\frac{1}{2}$ hours, what speed must they average for the remainder of their trip?
8. Four employees can wash 20 service vehicles in 5 hours. How long would it take 5 employees to wash the same number of vehicles?

9. These two figures are similar. Use a proportion to find the length of side n .

