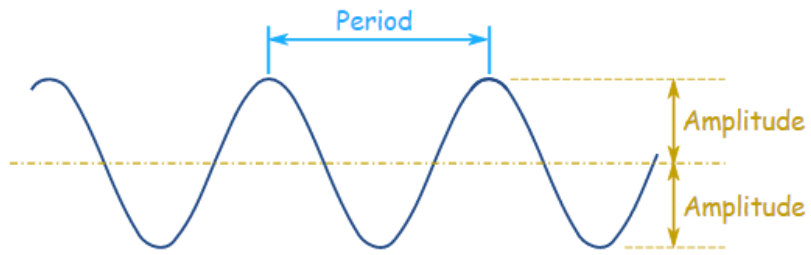
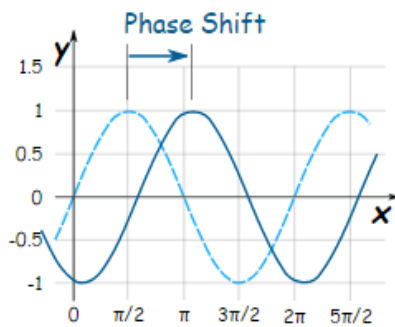


The **Period** goes from one peak to the next (or from any point to the next matching point):

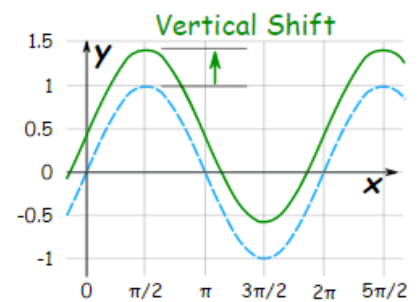


The **Amplitude** is the height from the center line to the peak (or to the trough). Or we can measure the height from highest to lowest points and divide that by 2.



The **Phase Shift** is how far the function is **horizontally** to the right of the usual position.

The **Vertical Shift** is how far the function is **vertically** up from the usual position.



We can have all of them in one equation:

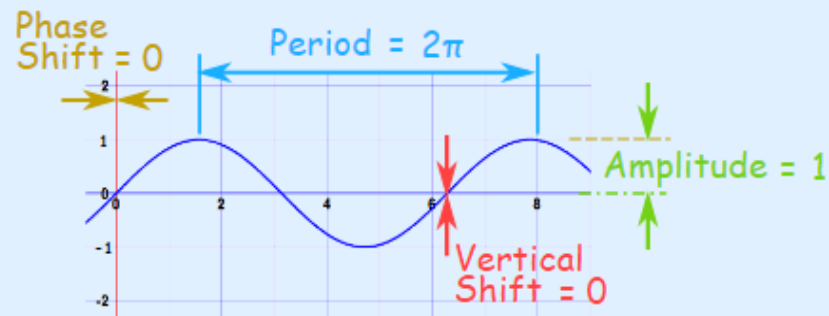
$$y = A \sin(Bx + C) + D$$

- amplitude is **A**
- period is  $2\pi/B$
- phase shift is  $-C/B$
- vertical shift is **D**

### Example: $\sin(x)$

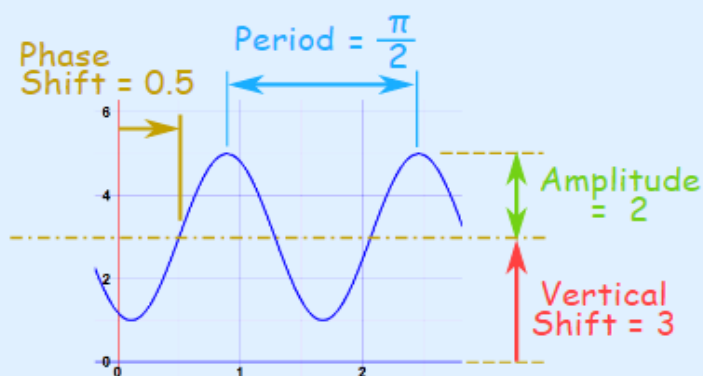
This is the basic unchanged sine formula. **A = 1, B = 1, C = 0 and D = 0**

So amplitude is **1**, period is  $2\pi$ , there is no phase shift or vertical shift:



### Example: $2 \sin(4x - 2) + 3$

- amplitude  $A = 2$
- period  $2\pi/B = 2\pi/4 = \pi/2$
- phase shift  $-C/B = -(-2)/4 = 1/2$
- vertical shift  $D = 3$



- the **2** tells us it will be 2 times taller than usual, so Amplitude = 2
- the usual period is  $2\pi$ , but in our case that is "sped up" (made shorter) by the **4** in  $4x$ , so Period =  $\pi/2$
- and it will be shifted to the right because of the  $-2$  (positive goes left, negative goes right), but because it is also "sped up" by **4** then it is shifted by only  **$1/2$** , so Phase Shift =  **$1/2$**
- lastly the  $+3$  tells us the center line is  $y = +3$ , so Vertical Shift = 3

State the amplitude, period, frequency, and phase shift of  $y = \sin\left(3x - \frac{\pi}{2}\right)$ . Then graph two periods of the function.

In this function,  $a = 1$ ,  $b = 3$ , and  $c = -\frac{\pi}{2}$ .

Amplitude:  $|a| = |1|$  or 1

Period:  $\frac{2\pi}{|b|} = \frac{2\pi}{|3|}$  or  $\frac{2\pi}{3}$

Frequency:  $\frac{|b|}{2\pi} = \frac{|3|}{2\pi}$  or  $\frac{3}{2\pi}$

Phase shift:  $-\frac{c}{|b|} = -\frac{-\frac{\pi}{2}}{|3|}$  or  $\frac{\pi}{6}$

To graph  $y = \sin\left(3x - \frac{\pi}{2}\right)$ , consider the graph of  $y = \sin 3x$ . The period of this function is  $\frac{2\pi}{3}$ . Create a table listing the coordinates of key points of  $y = \sin 3x$  on the interval  $\left[0, \frac{2\pi}{3}\right]$ .

To account for a phase shift of  $\frac{\pi}{6}$ , add  $\frac{\pi}{6}$  to the  $x$ -values of each of the key points for the graph of  $y = \sin 3x$ .

Function	$x$ -intercept	Maximum	$x$ -intercept	Minimum	$x$ -intercept
$y = \sin 3x$	$(0, 0)$	$\left(\frac{\pi}{6}, 1\right)$	$\left(\frac{\pi}{3}, 0\right)$	$\left(\frac{\pi}{2}, -1\right)$	$\left(\frac{2\pi}{3}, 0\right)$
$y = \sin\left(3x - \frac{\pi}{2}\right)$	$\left(\frac{\pi}{6}, 0\right)$	$\left(\frac{\pi}{3}, 1\right)$	$\left(\frac{\pi}{2}, 0\right)$	$\left(\frac{2\pi}{3}, -1\right)$	$\left(\frac{5\pi}{6}, 0\right)$

## Frequency

Frequency is how often something happens per unit of time (per "1").

