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3-4 Study Guide and Intervention
Exponential and Logarithmic Equations

Solve Exponential Equations & One-to-One Property of Exponential Functions: For $b > 0$ and $b \neq 1$, $bx = by$ if and only if $x = y$. This property will help you solve exponential equations. For example, you can express both sides of the equation as an exponent with the same base. Then use the property to set the exponents equal to each other and solve.
If the bases are not the same, you can *exponentiate* each side of an equation and use logarithms to solve the equation.

Example 1

a. Solve $4^{x-1} = 16^x$.

$4^{x-1} = 16^x$	Original equation
$4^{x-1} = (4^2)^x$	$16 = 4^2$
$4^{x-1} = 4^{2x}$	Power of a Power
$x - 1 = 2x$	One-to-One Property
$-1 = x$	Subtract *x* from each side.

b. Solve $e^{2x} - 3e^x + 2 = 0$.

$e^{2x} - 3e^x + 2 = 0$	Original equation
$u^2 - 3u + 2 = 0$	Write in quadratic form.
$(u - 2)(u - 1) = 0$	Factor.
$u = 2$ or $u = 1$	Solve.
$e^x = 2$ or $e^x = 1$	Substitute for *u*.
$x = \ln 2$ or 0	Take the natural logarithm of each side

Example 2: Solve each equation. Round to the nearest hundredth if necessary.

a. $3^x = 19$

$\log 3^x = \log 19$	Take the log of both sides.
$x \log 3 = \log 19$	Power Property
$x = \frac{\log 19}{\log 3}$	Divide each side by log 3.
$x \approx 2.68$	Use a calculator. Check this solution in the original equation.

b. $e^{8x+1} - 6 = 1$

$e^{8x+1} = 7$	Add 6 to both sides.
$\ln e^{8x+1} = \ln 7$	Take the ln of both sides.
$(8x + 1) \ln e = \ln 7$	Power Property
$8x + 1 = \ln 7$	$\ln e = 1$
$8x = \ln 7 - 1$	Subtract 1 from each side.
$x = \frac{\ln 7 - 1}{8} \approx 0.12$	Divide by 8 and use a calculator.

Exercises

Solve each equation. Round to the nearest hundredth.

1. $9^x = 3^{3x-4}$

2. $\left(\frac{1}{4}\right)^{2x-1} = \left(\frac{1}{8}\right)^{11-x}$

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3. $4^{3x-2} = \frac{1}{2}^{2x}$

4. $2e^{2x} + 12e^{x} - 54 = 0$

5. $9^{2x} = 12$

6. $2.4e^{x-6} = 9.3$

7. $3^{2x} = 6^{x-1}$

8. $e^{19x} = 23$

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3-4 Study Guide and Intervention *(continued)*

Exponential and Logarithmic Equations

Solve Logarithmic Equations **One-to-One Property of Exponential Functions**
For $b > 0$ and $b \neq 1$, $\log_b x = \log_b y$ if and only if $x = y$.

This property will help you solve logarithmic equations. For example, you can express both sides of the equation as a logarithm with the same base. Then convert both sides to exponential form, set the exponents equal to each other and solve.

Example 1: Solve $2 \log_5 4x - 1 = 11$.

$2 \log_5 4x - 1 = 11$	Original equation
$2 \log_5 4x = 12$	Add 1 to each side.
$\log_5 4x = 6$	Divide each side by 2.
$4x = 5^6$	Write in exponential form. (Use 5 as the base when exponentiating.)
$x = \frac{5^6}{4}$	Divide each side by 4.
$x = 3906.25$	Use a calculator.

Example 2: Solve $\log_2(x - 6) = 5 - \log_2 2x$.

$\log_2 (x - 6) = 5 - \log_2 2x$	Original equation
$\log_2 (x - 6) + \log_2 2x = 5$	Rearrange the logs.
$\log_2 (2x(x - 6)) = 5$	Product Property
$2x(x - 6) = 2^5$	Rewrite in exponential form.
$2x^2 - 12x - 32 = 0$	Expand.
$2(x - 8)(x + 2) = 0$	Factor.
$x = 8$ or -2	Solve.

CHECK

$x = -2$ $\log_2 (-2 - 6) = 5 - \log_2 [2(-2)]$ yields logs of negative numbers. Therefore, –2 is extraneous.

$x = 8$ $\log_2 (8 - 6) = 5 - \log_2 [2(8)]$

$\log_2 2 = 5 - \log_2 16$, which is true. Therefore, $x = 8$.

Exercises

Solve each logarithmic equation.

1. $\log 3x = \log 12$

2. $\log_{12} 2 + \log_{12} x = \log_{12} (x + 7)$

3. $\log (x + 1) + \log (x - 3) = \log (6x^2 - 6)$

4. $\log_3 3x = \log_3 36$

5. $\log (16x + 2) + \log (20x - 2) = \log (319x^2 + 9x - 2)$

6. $\ln x + \ln (x + 16) = \ln 8 + \ln (x + 6)$