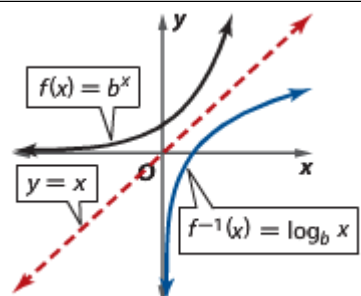


### Section 3.2– Logarithmic Functions

## LOGARTIHMIC FUNCTIONS

*Objective: Evaluate expressions involving logarithms*

<b>LOGARITHMIC FUNCTION</b>	
<p><i>Definition:</i></p>	
<p><i>Relating Logarithmic and Exponential Functions</i></p>	

**EXAMPLE 1:** Evaluate Logarithms

Evaluate each logarithm.

a)  $\log_3 81$

b)  $\log_7 \frac{1}{49}$

c)  $\log_5 \sqrt{5}$

d)  $\log_2 2$

e)  $\log_8 512$

f)  $\log_4 4^{3.2}$

g)  $\log_2 \frac{1}{32}$

h)  $\log_{16} \sqrt{2}$

<b>Basic Properties of Logs</b>			
<p>If <math>b &gt; 0</math>, <math>b \neq 1</math>, and <math>x</math> is a real number, then the following statements are true:</p>			

*EXAMPLE 2: Apply Properties of Logarithms*

Evaluate each expression.

a)  $\log_5 125$

b)  $12^{\log_{12} 4.7}$

c)  $\log_9 81$

d)  $3^{\log_3 1}$

Common Logarithm			
Basic Properties of Common Logs			
If $x$ is a real number, then the following statements are true:			

*EXAMPLE 3: Common Logarithms*

Evaluate each expression.

a)  $\log 0.001$

b)  $10^{\log 5}$

c)  $\log 26$

d)  $\log(-5)$

e)  $\log 10,000$

f)  $\log 0.081$

g)  $\log -0$

h)  $10^{\log 3}$

Natural Logarithm			
Basic Properties of Natural Logs			
If $x$ is a real number, then the following statements are true:			

*EXAMPLE 4: Natural Logarithms*

Evaluate each expression.

a)  $\ln e^{0.73}$

b)  $\ln(-5)$

c)  $e^{\ln 6}$

d)  $\ln 4$

e)  $\ln 32$

f)  $e^{\ln 4}$

g)  $\ln\left(\frac{1}{e^3}\right)$

h)  $-\ln 9$

## GRAPHS OF LOGARITHMIC FUNCTIONS

*Objective: Sketch and analyze graphs of logarithmic functions*

*EXAMPLE 5: Graphs of Logarithmic Functions*

Sketch and analyze the graph of each function. Describe its domain, range, intercepts, asymptotes, end behavior, and where the function is increasing or decreasing.

a)  $f(x) = \log_3 x$

Domain:

Range:

y-intercept:

Asymptote:

What is  $f^{-1}(x)$ ?

end behavior:

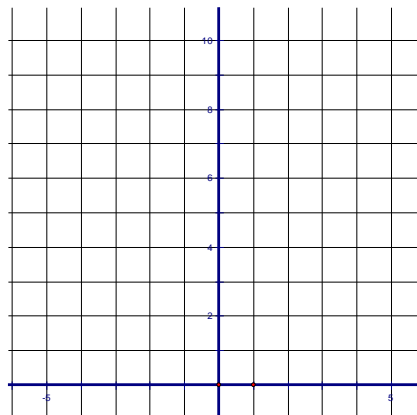
Construct a table for  $f^{-1}(x)$

increasing:

decreasing:

$x$	-4	-2	-1	0	1	2
$f^{-1}(x)$						

$f^{-1}(x)$						
$x$						



b)  $f(x) = \log_{\frac{1}{2}} x$

What is  $f^{-1}(x)$ ?

Construct a table for  $f^{-1}(x)$

$x$	-4	-2	-1	0	1	2
$f^{-1}(x)$						

$f^{-1}(x)$						
$x$						

Domain:

Range:

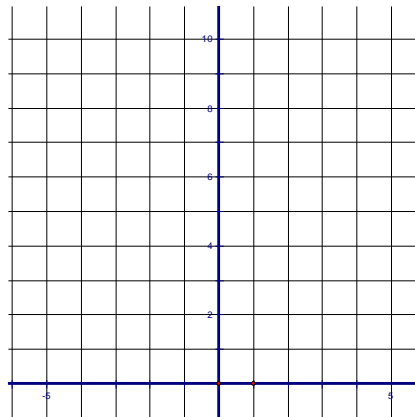
y-intercept:

Asymptote:

end behavior:

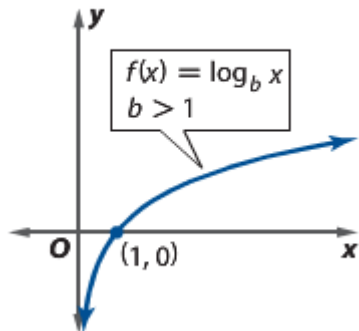
increasing:

decreasing:



**PROPERTIES OF LOGARITHMIC FUNCTIONS**

**Logarithmic Growth**



Domain:

Range:

y-intercept:

x- intercept:

end behavior:

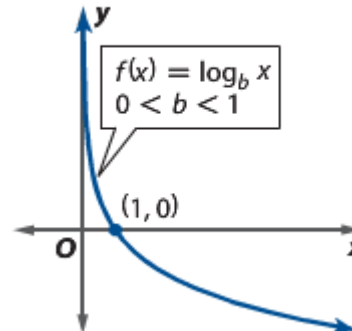
increasing:

decreasing:

asymptote:

continuity:

**Logarithmic Decay**



Domain:

Range:

y-intercept:

x- intercept:

end behavior:

increasing:

decreasing:

asymptote:

continuity:

## Transformations of functions

	Shift left 2	Shift right 2	Shift up 2	Shift down 2	Reflect over x-axis	Reflect over y-axis
Function Notation						
$f(x) = \log_3 x$						

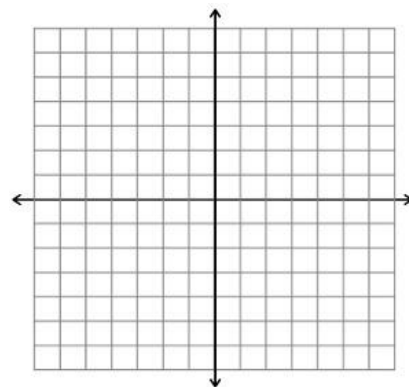
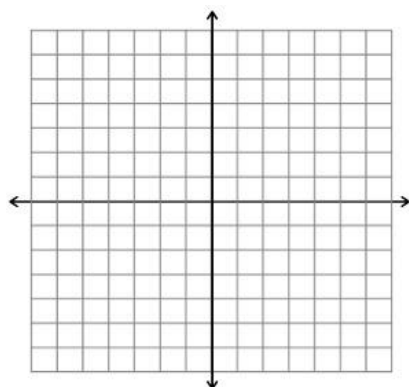
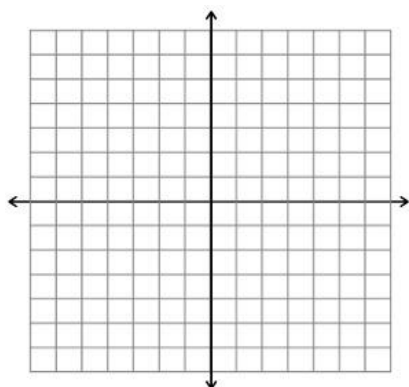
### EXAMPLE 6: Graph Transformations of Logarithmic Functions

Use the graph of  $f(x) = \log x$  to describe the transformation that results in each function. Then sketch the graph of the functions.

a)  $f(x) = \log(x+4)$

b)  $f(x) = -\log x - 5$

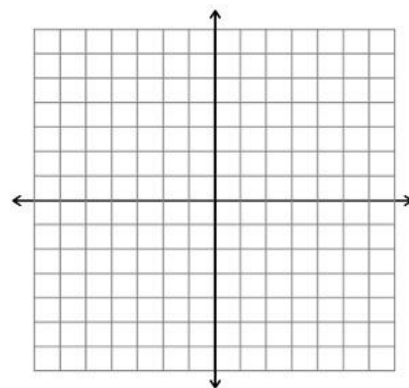
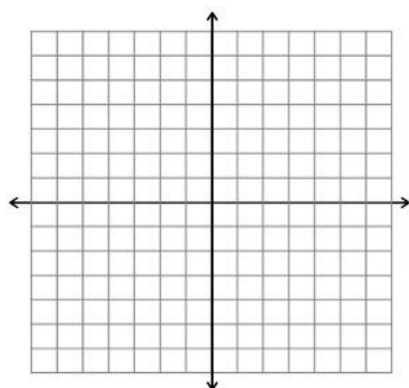
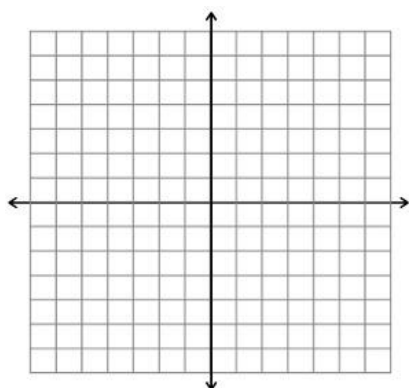
c)  $f(x) = 3\log(x+2)$



d)  $f(x) = \ln(x-6)$

e)  $f(x) = 0.5\ln x - 2$

f)  $f(x) = \ln(x+4) + 3$



*EXAMPLE 7: Use Logarithmic Functions*

- a) The intensity level of a sound, measured in decibels, can be modeled by  $d(w) = 10 \log \frac{w}{w_0}$  where  $w$  is the intensity of the sound in watts per square meter and  $w_0$  is the constant  $1.0 \times 10^{-12}$  watts per square meter.
- i. If the intensity of the sound of a person talking loudly is  $3.16 \times 10^{-8}$  watts per square meter, what is the intensity level of a sound in decibels?
  - ii. If the threshold for hearing for a certain person with hearing loss is 5 decibels, will a sound with an intensity level  $2.1 \times 10^{-12}$  watts per square meter be audible to that person?
  - iii. Sounds in excess of 85 decibels can cause hearing damage. Determine the intensity of a sound with an intensity level of 85 decibels.
- b) The number of machines infected by a specific computer virus can be modeled by  $c(d) = 6.8 + 20.1 \ln d$ , where  $d$  is the number of days since the first machine was infected.
- i. About how many machines were infected on day 12?
  - ii. How many more machines were infected on day 30 than on day 12?
  - iii. On about what day will the number of infected machines reach 75?