2.4 Zeros of Polynomial Equations

Objectives:

- Find real zeros of polynomial functions
- Find complex zeros of polynomial functions

Real Zeros:

Rational Zeros - can be expressed as a ratio of two integers

Irrational Zeros - can not be expressed as a ratio of two integers

The Rational Zero Theorem

RZT : If p/q is a zero of a polynomial equation, then p is a factor of the constant term, and q is a factor of the highest degree term's coefficient.

$$f(x) = 12x^5 - 9x^4 + 17x + 26$$

leading coef.

constant term

Find all the possible rational roots:

1. $f(x) = x^3 + 3x^2 - x + 8$

2. $f(x) = 2x^4 - 5x^3 + 8x^2 + 3x - 5$

3. $f(x) = 5x^6 - 3x^4 + 5x^3 + 2x^2 - 15$

Descartes' Rule of Signs

(for finding the # and type of roots)

R^+ **Positive Real Roots** - the number of the sign change of $p(x)$, or this # less an even value.

R^- **Negative Real Roots** - the number of the sign change of $p(-x)$, or this # less an even value.

I **Imaginary Roots** : Degree - $(R^+ + R^-)$

State the number and type of roots:

1. $f(x) = 3x^3 + x^2 - 8x - 12$

2. $f(x) = 3x^5 - x^4 - x^3 + 6x^2 - 5$

3. $f(x) = 10x^{10} - 3x^8 + 4x^6 - 2x^4 + x^2 - 3$

Find all the rational zeros:

1. $f(x) = x^3 - 3x^2 - 4x + 12$

2. $f(x)=3x^3-9x^2-10x-8$

3. $f(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$

Write a polynomial of least degree with the given zeros:

1. 2, 1, -4

2. -2, multiplicity 2 and $3i$

3. $-2, 3, 2+i$

4. $4, \sqrt{3}$

Find all zeros:

Describe the real zeros using Descartes' Rule. List the Possible Rational Zeros. Find all zeros.

ex. 1 $f(x) = 2x^4 + 3x^3 + 6x^2 + 12x - 8$

Use the given zeros to find all the complex zeros of the function:

Ex. 1 $f(x) = 2x^4 + 3x^3 + 6x^2 + 12x - 8;\ \ 2i$

Ex. 2 $f(x) = x^3 - 2x + 4; \quad 1 - i$

Assignment:

W.S. 2.4