2.2 Polynomial Functions

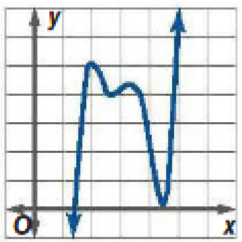
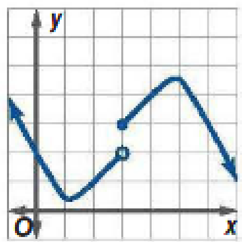
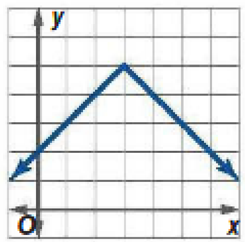
OBJECTIVES:

- Graph polynomial functions.

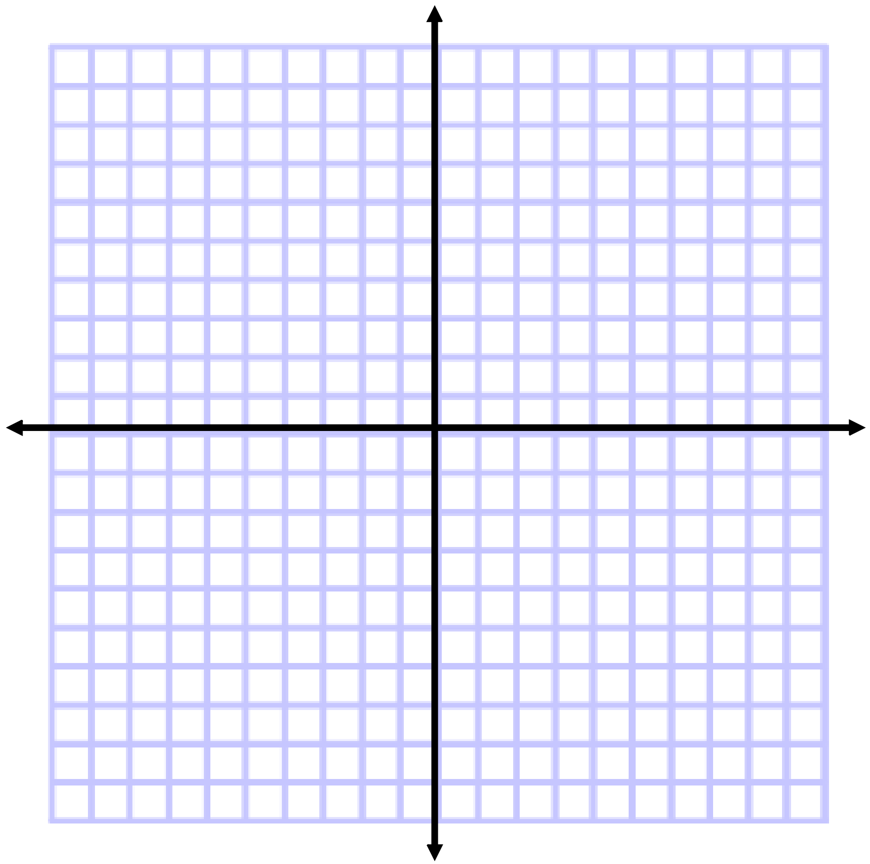
Let n be a nonnegative integer and let $a_0, a_1, a_2, \ldots, a_{n-1}, a_n$ be real numbers with $a_n \neq 0$. Then the function given by

$$f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

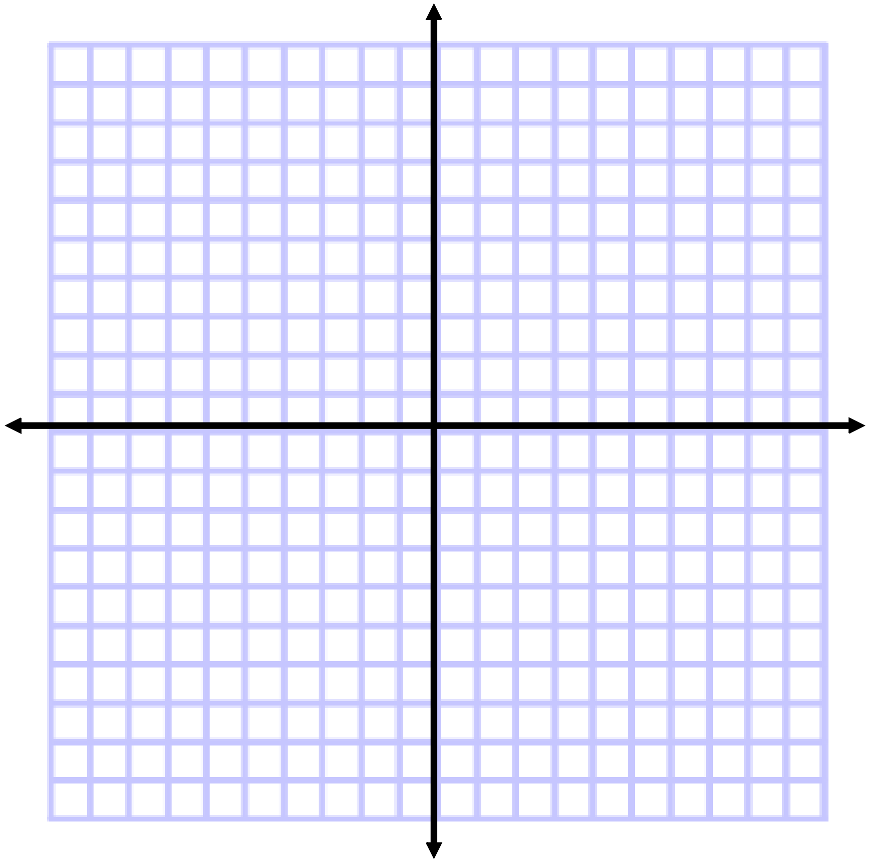
is called a **polynomial function of degree n**. The **leading coefficient** of a polynomial function is the coefficient of the variable with the greatest exponent. The leading coefficient of $f(x)$ is a_n.

Graphs of Polynomial Functions	
Example	**Nonexamples**
	 
Polynomial functions are defined and continuous for all real numbers and have smooth, rounded turns.	Graphs of polynomial functions do not have breaks, holes, gaps, or sharp corners.

Example 1: $f(x) = (x - 2)^5$

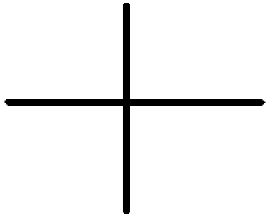


Example 2: $g(x) = -x^4 + 3$



Leading Term Test for Polynomial End Behavior

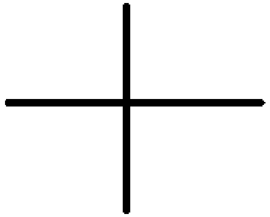
even degree with a positive leading coefficient



$$\lim_{x\to\infty} f(x) =$$

$$\lim_{x\to-\infty} f(x) =$$

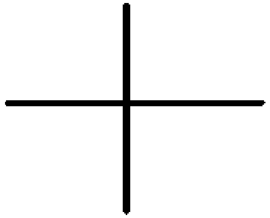
even degree with a negative leading coefficient



$$\lim_{x\to\infty} f(x) =$$

$$\lim_{x\to-\infty} f(x) =$$

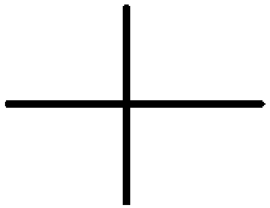
odd degree with a positive leading coefficient



$$\lim_{x\to\infty} f(x) =$$

$$\lim_{x\to-\infty} f(x) =$$

odd degree with a negative leading coefficient



$$\lim_{x\to\infty} f(x) =$$

$$\lim_{x\to-\infty} f(x) =$$

Describe the end behavior of the graph of each polynomial function using limits. Explain your reasoning using the leading term test.

Example 3: $f(x) = 4x^5 - 8x^3 + 20$

$$\lim_{x \to \infty} f(x) =$$

$$\lim_{x \to -\infty} f(x) =$$

Example 4: $g(x) = -2x^6 + 11x^4 + 2x^2$

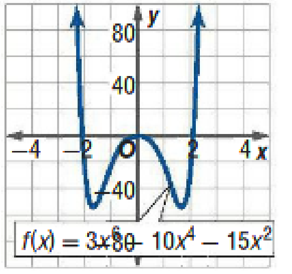
$$\lim_{x \to \infty} f(x) =$$

$$\lim_{x \to -\infty} f(x) =$$

KeyConcept Zeros and Turning Points of Polynomial Functions

A polynomial function f of degree $n \geq 1$ has at most n distinct real zeros and at most $n - 1$ turning points.

Example Let $f(x) = 3x^6 - 10x^4 - 15x^2$. Then f has at most 6 distinct real zeros and at most 5 turning points. The graph of f suggests that the function has 3 real zeros and 3 turning points.



Recall that if f is a polynomial function and c is an x-intercept of the graph of f, then it is equivalent to say that:

- c is a zero of f,
- $x = c$ is a solution of the equation $f(x) = 0$, and
- $(x - c)$ is a factor of the polynomial $f(x)$.

Zeros of a Polynomial Function:

State the number of possible real zeros and turning points of the given function. Find the real zeros by factoring.

Example 5: $f(x) = x^3 - 6x^2 - 27x$

possible real zeros:______

real zeros: ______

possible turning points: ______

Example 6: $g(x) = x^4 - 8x^2 + 15$

possible real zeros:______

real zeros: ______

possible turning points: ______

Example 7: $h(x) = x^5 - 6x^3 - 16x$

possible real zeros:______

real zeros: ______

possible turning points: ______

Polynomial Functions with Repeated Zeros:

Example 8: $h(x) = 3x^5 - 18x^4 + 27x^3$

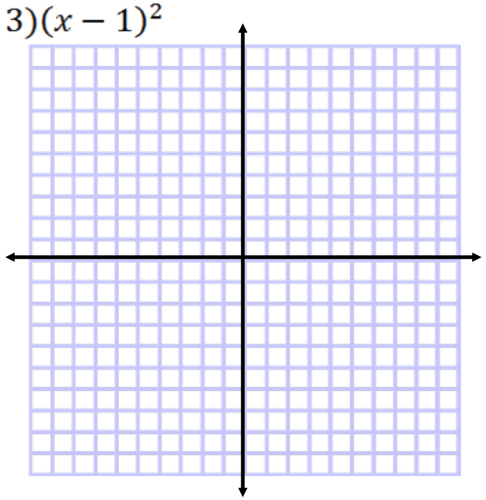
possible real zeros:______

real zeros: ______

possible turning points: ______

For the function, apply the leading term test, determine the zeros and state the multiplicity of any zero, find a few additional points, then graph the function

Example 9: $f(x) = x(2x+3)(x-1)^2$



If a factor $(x - c)$ occurs more than once in the completely factored form of $f(x)$, then its related zero c is called a **repeated zero**. When the zero occurs an even number of times, the graph will be tangent to the x-axis at that point. When the zero occurs an odd number of times, the graph will cross the x-axis at that point. A graph is tangent to an axis when it touches the axis at that point, but does not cross it.

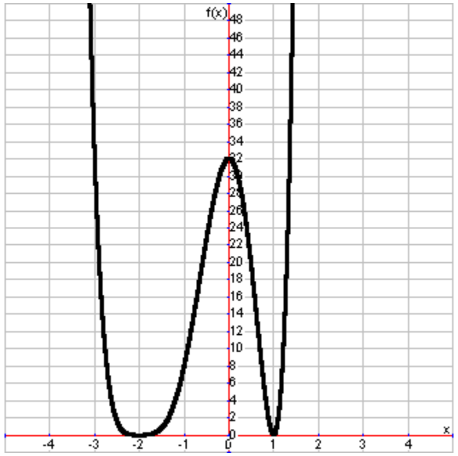
KeyConcept Repeated Zeros of Polynomial Functions

If $(x - c)^m$ is the highest power of $(x - c)$ that is a factor of polynomial function f, then c is a zero of **multiplicity** m of f, where m is a natural number.

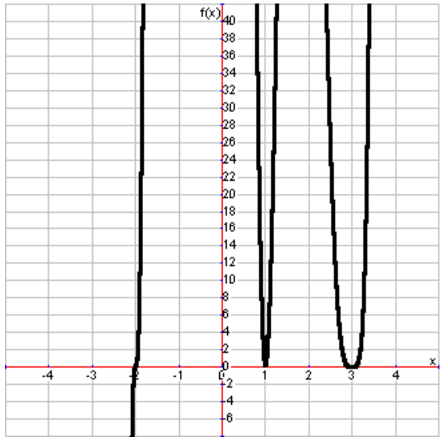
- If a zero c has odd multiplicity, then the graph of f crosses the x-axis at $x = c$ and the value of $f(x)$ changes signs at $x = c$.
- If a zero c has even multiplicity, then the graph of f is tangent to the x-axis at $x = c$ and the value of $f(x)$ does not change signs at $x = c$.

Notice the end behavior and roots:

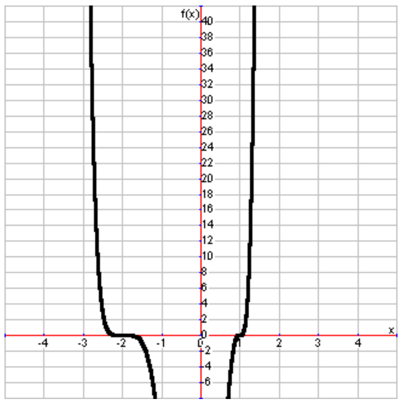
$$f(x)=2(x-1)^2(x+2)^4$$



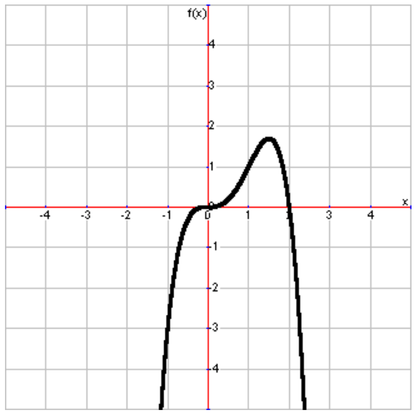
$$f(x)=2(x-1)^2(x+2)^3(x-3)^4$$



$$f(x)=2(x-1)^3(x+2)^5$$

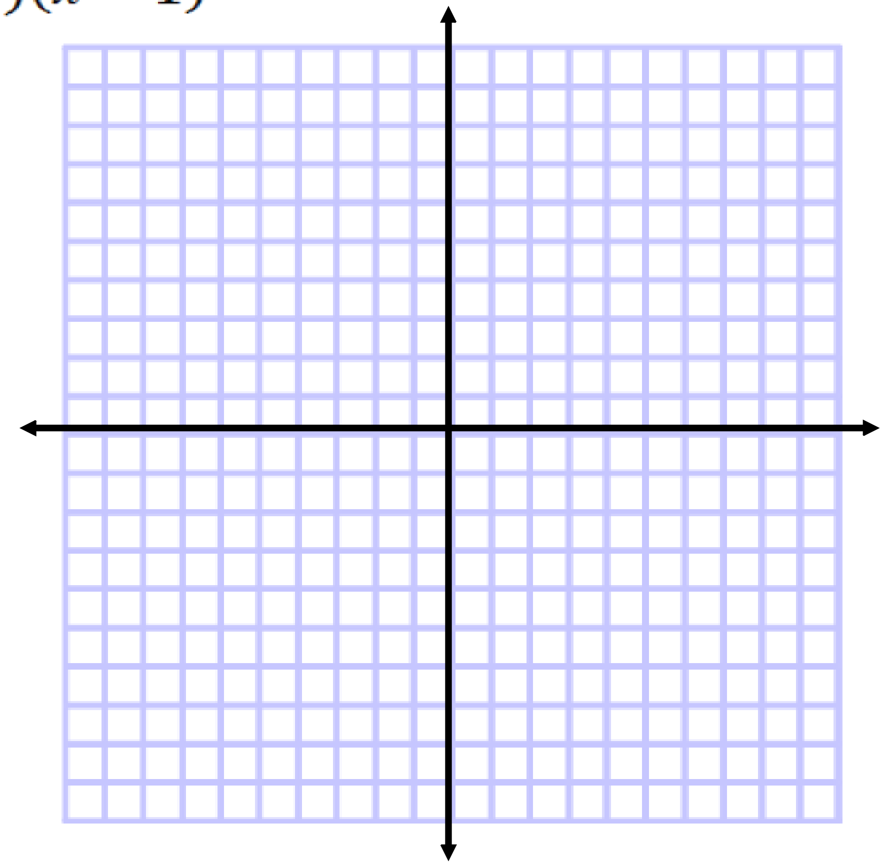


$$f(x)=-x^3(x-2)$$



For the function, apply the leading term test, determine the zeros and state the multiplicity of any zero, find a few additional points, then graph the function

Example 9: $f(x) = x(2x + 3)(x - 1)^2$



Write a function of least degree whose zeros are given:

Example 10: 3, -2, 3/2

Assignment:

pg. 104 (1-41 odd, 69, 70)