

## 1.7 Inverse Relations and Functions

OBJECTIVES:

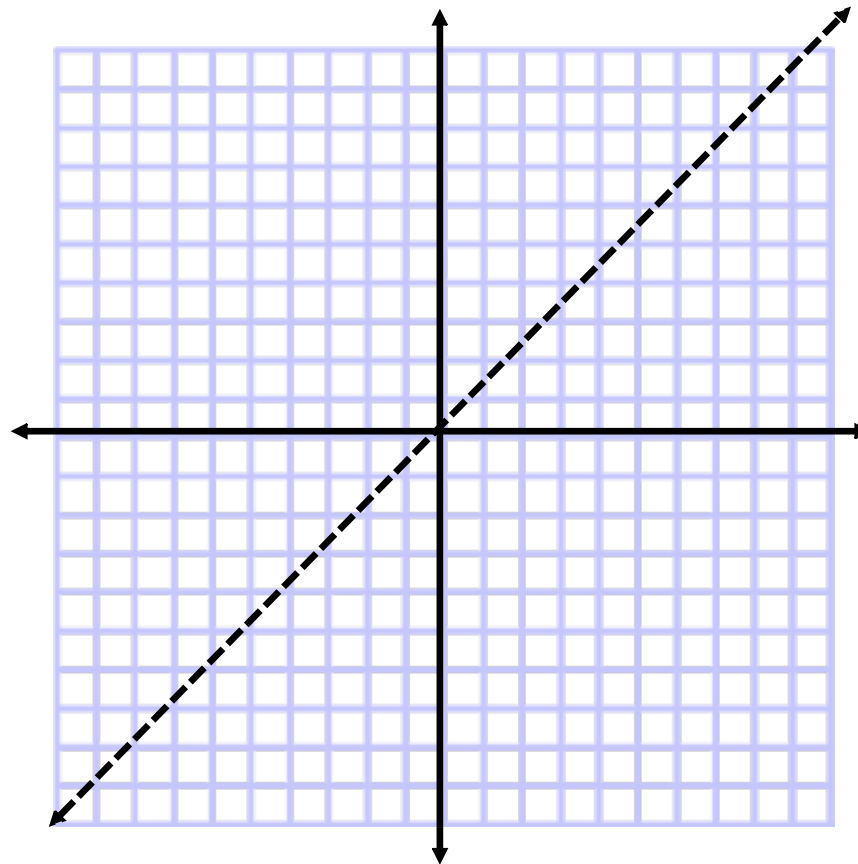
- Use the horizontal line test to determine inverse functions.
- Find inverse functions algebraically and graphically

**Inverse Relations:** Inverse relations exist if and only if one relation contains  $(b,a)$  whenever the other relation contains  $(a,b)$ .

Comparing a relation and its inverse:

$$x = y^2 - 4 \text{ or } y^2 = x + 4$$

x	y
5	-3
0	-2
-3	-1
-4	0
-3	1
0	2
5	3



$$y = x^2 - 4$$

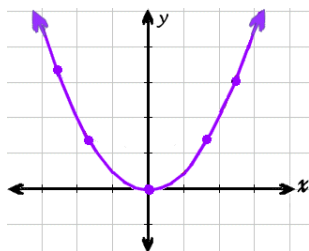
x	y
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5

# 1.7 Inverse Functions.notebook

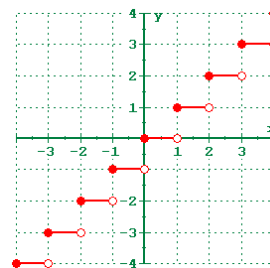
**Horizontal Line Test:** A function  $f$  has an inverse  $f^{-1}$  iff each horizontal line intersects the graph of the function in at most one point.

Do the following functions have functional inverses?

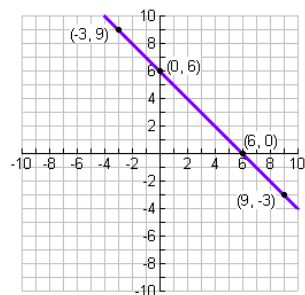
ex. 1



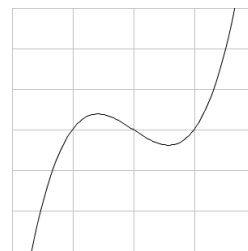
ex. 2



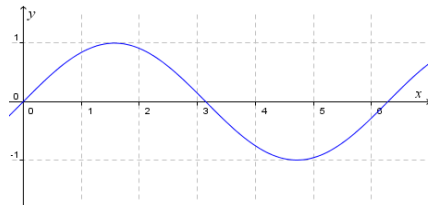
ex. 3



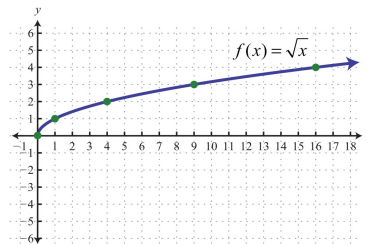
ex. 4



ex. 5



ex. 6

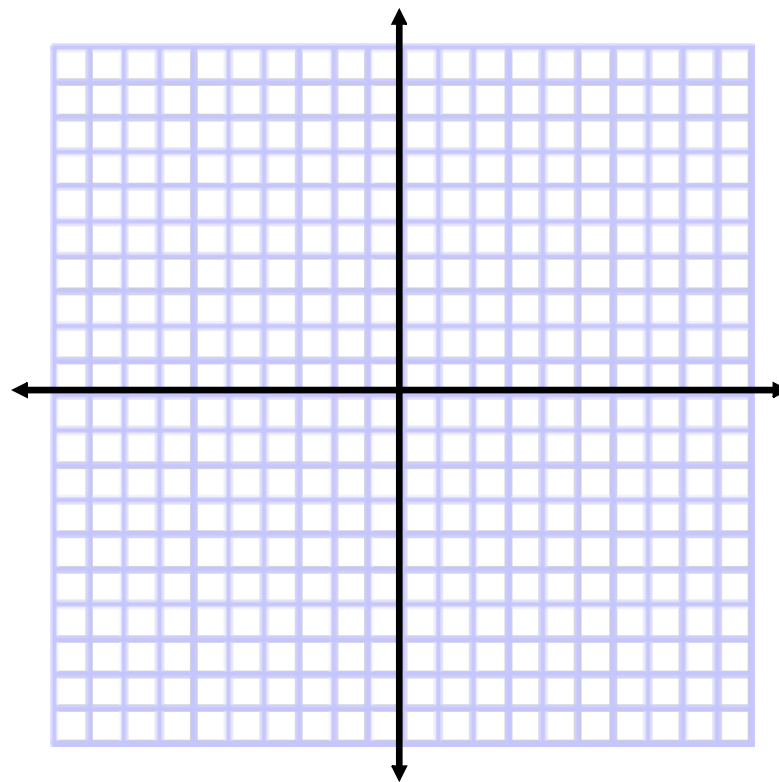


Does an inverse function exist? (You should not need a graphing calc.)

ex. 1  $y = -(x-4)^3 + 1$

ex. 2  $y = \frac{1}{x+2}$

ex. 3  $y = \sqrt{-x} + 3$



Use a graphing calc. to determine whether or not an inverse function exists:

ex. 1  $h(x) = \frac{4}{x}$

ex. 2  $f(x) = x^2 + 5x - 7$

**one-to-one:** A function that passes the horizontal line test. In other words, no  $x$ -value is matched with more than one  $y$ -value and no  $y$ -value is matched with more than one  $x$ .

## Finding Inverse Functions

- Determine whether the function has an inverse by checking to see if it is one-to-one using the horizontal line test.
- swap the  $y$  and  $x$
- solve for  $y$

## 1.7 Inverse Functions.notebook

Find the inverse, if it exists, algebraically. Compare the domain and range of the original function to that of its inverse.

$$\boxed{\text{ex. 1}} \quad f(x) = \frac{x - 1}{x + 2}$$

## 1.7 Inverse Functions.notebook

$$\boxed{\text{ex. 2}} \quad f(x) = \sqrt{x - 4}$$



## Compositions of Inverse Functions

Two functions  $f$  and  $g$  are inverse functions iff:

\_\_\_\_\_ and \_\_\_\_\_

### Verifying Inverse Functions:

**ex. 1** Show that the functions are inverse functions

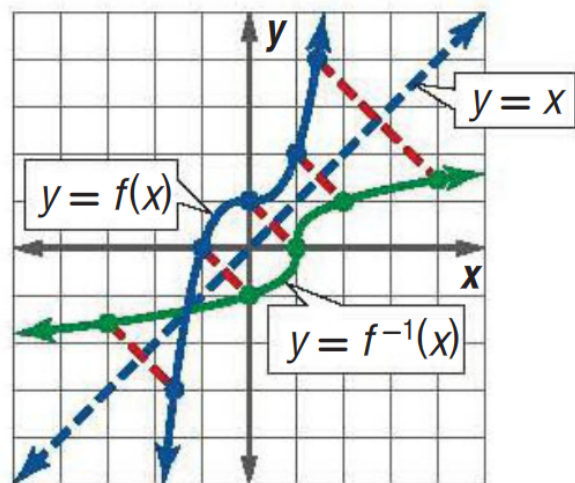
$$f(x) = 18 - 3x \quad g(x) = 6 - \frac{x}{3}$$

## 1.7 Inverse Functions.notebook

$$\boxed{\text{ex. 2}} \quad f(x) = x^2 + 10, x \geq 0; g(x) = \sqrt{x - 10}$$

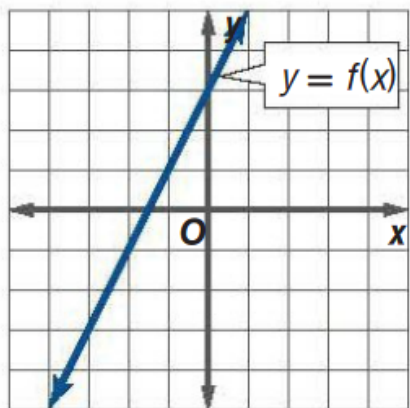
## Finding Inverse Functions Graphically:

If given one graph, you can produce the inverse by reflecting over the line  $y = x$ .

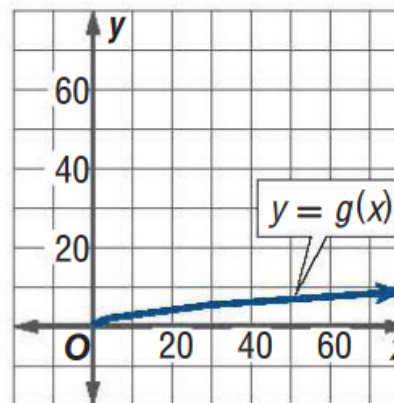


Use the graph of each function to graph its inverse function:

ex. 1



ex. 2



## 1.7 Inverse Functions.notebook

Assignment:

pg. 70 (1-35, 39, 43, 47-53) odd only