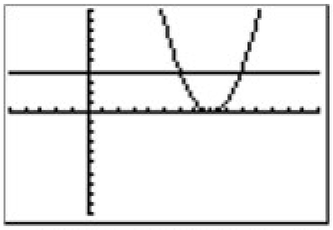
1-7 Inverse Relations and Functions

Graph each function using a graphing calculator, and apply the horizontal line test to determine whether its inverse function exists. Write *yes* or *no*.

2. $f(x) = x^2 - 16x + 64$

SOLUTION:

The graph of $f(x) = x^2 - 16x + 64$ below shows that it is possible to find a horizontal line that intersects the graph of $f(x)$ more than once. Therefore, you can conclude that an inverse function does not exist.



[−5, 15] scl: 1 by [−10, 10] scl: 1

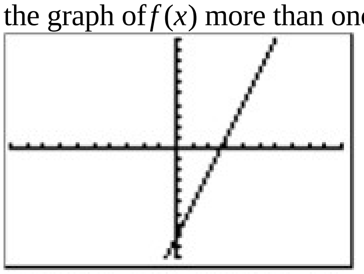
ANSWER:

no

4. $f(x) = 3x - 8$

SOLUTION:

It appears from the portion of the graph of $f(x) = 3x - 8$ shown below that there is no horizontal line that intersects the graph of $f(x)$ more than once. Therefore, you can conclude that an inverse function does exist.



[−10, 10] scl: 1 by [−10, 10] scl: 1

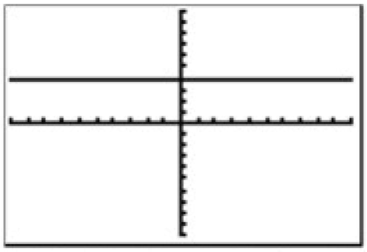
ANSWER:

yes

6. $f(x) = 4$

SOLUTION:

The graph of $f(x) = 4$ below shows that it is possible to find a horizontal line that intersects the graph of $f(x)$ more than once. That horizontal line is the same as the graph of $f(x)$ itself. Therefore, you can conclude that an inverse function does not exist.



[−10, 10] scl: 1 by [−10, 10] scl: 1

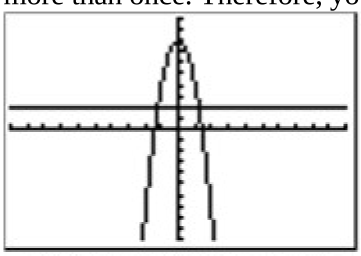
ANSWER:

no

8. $f(x) = -4x^2 + 8$

SOLUTION:

The graph of $f(x) = -4x^2 + 8$ below shows that it is possible to find a horizontal line that intersects the graph of $f(x)$ more than once. Therefore, you can conclude that an inverse function does not exist.



[−10, 10] scl: 1 by [−10, 10] scl: 1

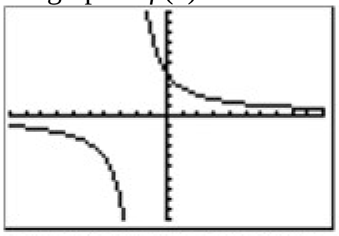
ANSWER:

no

10. $f(x) = \frac{8}{x+2}$

SOLUTION:

It appears from the portion of the graph of $f(x) = \frac{8}{x+2}$ shown below that there is no horizontal line that intersects the graph of $f(x)$ more than once. Therefore, you can conclude that an inverse function does exist.



[−10, 10] scl: 1 by [−10, 10] scl: 1

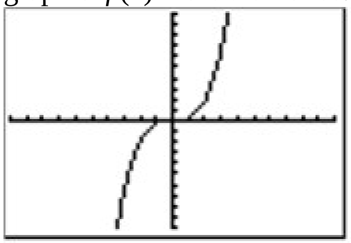
ANSWER:

yes

12. $f(x) = \frac{1}{4}x^3$

SOLUTION:

It appears from the portion of the graph of $f(x) = \frac{1}{4}x^3$ shown below that there is no horizontal line that intersects the graph of $f(x)$ more than once. Therefore, you can conclude that an inverse function does exist.



[−10, 10] scl: 1 by [−10, 10] scl: 1

ANSWER:

yes

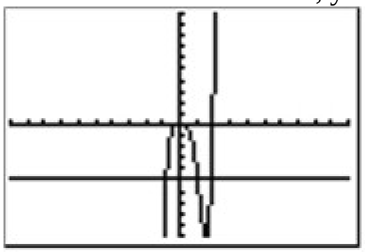
1-7 Inverse Relations and Functions

Determine whether each function has an inverse function. If it does, find the inverse function and state any restrictions on its domain.

14. $f(x) = 4x^5 - 8x^4$

SOLUTION:

The graph of $f(x) = 4x^5 - 8x^4$ below shows that it is possible to find a horizontal line that intersects the graph of $f(x)$ more than once. Therefore, you can conclude that an inverse function does not exist.



[−10, 10] scl: 1 by [−10, 10] scl: 1

ANSWER:

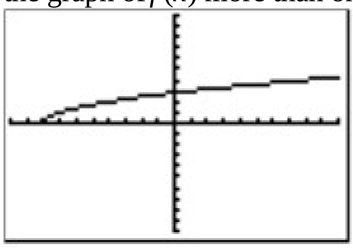
no

1-7 Inverse Relations and Functions

16. $f(x) = \sqrt{x+8}$

SOLUTION:

It appears from the portion of the graph of $f(x) = \sqrt{x+8}$ shown below that there is no horizontal line that intersects the graph of $f(x)$ more than once. Therefore, you can conclude that an inverse function does exist.

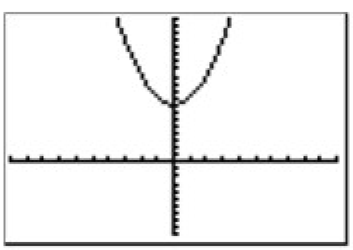


[–10, 10] scl: 1 by [–10, 10] scl: 1

The function f has domain $[-8, \infty)$ and range $[0, \infty)$.

$$y = \sqrt{x-8}$$
$$x = \sqrt{y-8}$$
$$x^2 = y - 8$$
$$x^2 + 8 = y$$
$$f^{-1}(x) = x^2 - 8$$

From the graph of $y = x^2 - 8$ below, you can see that the inverse relation has domain $(-\infty, \infty)$ and range $[8, \infty)$.



[–10, 10] scl: 1 by [–10, 20] scl: 1

By restricting the domain of the inverse relation to $[0, \infty)$, the domain and range of f are equal to the range and domain of f^{-1}, respectively. Therefore, $f^{-1}(x) = x^2 - 8$ for $x \geq 0$.

ANSWER:

yes; $f^{-1}(x) = x^2 - 8, x \geq 0$

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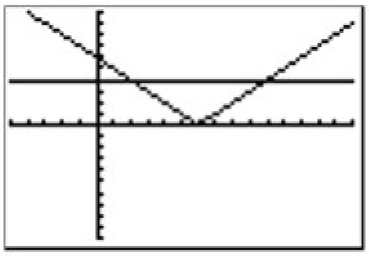
18. $f(x) = | x - 6 |$

SOLUTION:

The graph of $f(x) = | x - 6 |$ below shows that it is possible to find a horizontal line that intersects the graph of $f(x)$ more than once. Therefore, you can conclude that an inverse function does not exist.



[–5, 15] scl: 1 by [–10, 10] scl: 1

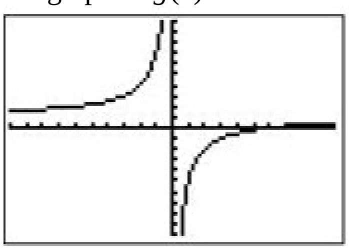
ANSWER:

no

20. $g(x) = \frac{x-6}{x}$

SOLUTION:

It appears from the portion of the graph of $g(x) = \frac{x-6}{x}$ shown below that there is no horizontal line that intersects the graph of $g(x)$ more than once. Therefore, you can conclude that an inverse function does exist.



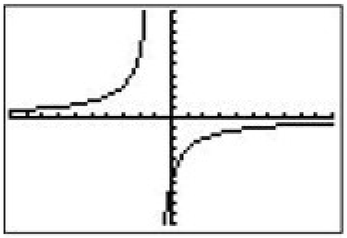
[–10, 10] scl: 1 by [–10, 10] scl: 1

The function g has domain $(-\infty, 0) \cup (0, \infty)$ and range $[-\infty, 1) \cup (1, \infty)$.

$$\begin{aligned} y &= \frac{x-6}{x} \\ x &= \frac{y-6}{y} \\ xy &= y-6 \\ xy + y &= -6 \\ y(x+1) &= -6 \\ y &= \frac{-6}{x+1} \end{aligned}$$

$g^{-1}(x) = \frac{-6}{x-1}$

From the graph $y = \frac{-6}{x-1}$ below, you can see that the inverse relation has domain $[-\infty, 1) \cup (1, \infty)$ and range $(-\infty, 0) \cup (0, \infty)$.



[–10, 10] scl: 1 by [–10, 10] scl: 1

The domain and range of g are equal to the range and domain of g^{-1}, respectively. Therefore, no further restrictions are necessary. $g^{-1}(x) = \frac{-6}{x-1}$ for $x \neq 1$.

ANSWER:

yes; $g^{-1}(x) = \frac{-6}{x-1}$, $x \neq 1$

Show algebraically that f and g are inverse functions.

28. $f(x) = 4x + 9$

$g(x) = \dfrac{x-9}{4}$

SOLUTION:

$f(x)$ and $g(x)$ are inverses if $f[g(x)] = g[f(x)] = x$.

Find each composition to show that $f(x)$ and $g(x)$ are inverses.

$$\begin{aligned} f[g(x)] &= 4\left(\frac{x-9}{4}\right)+9 \\ &= x-9+9 \\ &= x \\ g[f(x)] &= \frac{4x+9-9}{4} \\ &= \frac{4x}{4} \\ &= x \end{aligned}$$

ANSWER:

$$\begin{aligned} f[g(x)] &= 4\left(\frac{x-9}{4}\right)+9 \\ &= x-9+9 \\ &= x \\ g[f(x)] &= \frac{4x+9-9}{4} \\ &= \frac{4x}{4} \\ &= x \end{aligned}$$

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29. $f(x) = -3x^2 + 5;\ x \geq 0$

$g(x) = \sqrt{\frac{5-x}{3}}$

SOLUTION:

$f(x)$ and $g(x)$ are inverses if $f[g(x)] = g[f(x)] = x$.

Find each composition to show that $f(x)$ and $g(x)$ are inverses.

$$\begin{aligned} f[g(x)] &= -3\left(\sqrt{\frac{5-x}{3}}\right)^2 + 5 \\ &= \frac{-3(5-x)}{3} + 5 \\ &= x - 5 + 5 \\ &= x \end{aligned}$$

$$\begin{aligned} g[f(x)] &= \sqrt{\frac{5-(-3x^2+5)}{3}} \\ &= \sqrt{\frac{3x^2}{3}} \\ &= \sqrt{x^2} \\ &= x \end{aligned}$$

Note that the domain restriction is needed because $\sqrt{x^2} = \pm x$.

ANSWER:

$$\begin{aligned} f[g(x)] &= -3\left(\sqrt{\frac{5-x}{3}}\right)^2 + 5 \\ &= \frac{-3(5-x)}{3} + 5 \\ &= x - 5 + 5 \\ &= x \end{aligned}$$

$$\begin{aligned} g[f(x)] &= \sqrt{\frac{5-(-3x^2+5)}{3}} \\ &= \sqrt{\frac{3x^2}{3}} \\ &= \sqrt{x^2} \\ &= x \end{aligned}$$

30. $f(x) = \frac{x^2}{4} + 8;\ x \geq 0$
$g(x) = \sqrt{4x - 32}$

SOLUTION:

$f(x)$ and $g(x)$ are inverses if $f[g(x)] = g[f(x)] = x$.

Find each composition to show that $f(x)$ and $g(x)$ are inverses.

$$\begin{aligned} f[g(x)] &= \frac{\left(\sqrt{4x-32}\right)^2}{4} + 8 \\ &= \frac{4x-32}{4} + 8 \\ &= x - 8 + 8 \\ &= x \\ g[f(x)] &= \sqrt{4\left(\frac{x^2}{4} + 8\right) - 32} \\ &= \sqrt{x^2 + 32 - 32} \\ &= \sqrt{x^2} \\ &= x \end{aligned}$$

Note that the domain restriction is needed because $\sqrt{x^2} = \pm x$.

ANSWER:

$$\begin{aligned} f[g(x)] &= \frac{\left(\sqrt{4x-32}\right)^2}{4} + 8 \\ &= \frac{4x-32}{4} + 8 \\ &= x - 8 + 8 \\ &= x \\ g[f(x)] &= \sqrt{4\left(\frac{x^2}{4} + 8\right) - 32} \\ &= \sqrt{x^2 + 32 - 32} \\ &= \sqrt{x^2} \\ &= x \end{aligned}$$